Performance of a Forward-Acting Error-Control System on the Switched Telephone Network

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This brief contains a summary of data taken in the course of a recent error-control experiment. In this experiment data were transmitted over switched voiceband telephone lines at 2000 bits per second using *Data-Phone** data set 201A. With the transmitting terminal located at the Holmdel, New Jersey laboratory, connections via the switched network were established to various cities (Baltimore, Cleveland, Dallas, Denver, Louisville, and St. Louis). There a return connection, again via the switched network, was established to the receiving terminal which was also located at Holmdel. Nearly all calls were made during the business day.

Errors were corrected by means of a forward-acting cyclic code which was formed by interleaving the (15,9) code generated by $x^6 + x^5 + x^4 + 1$ to degree i. As a result, $(9/15) \cdot 2000 = 1200$ information bits per second were transmitted. In the first half of the experiment, i was set to 73; in the second half, 200. Since the (15,9) code corrects all bursts of length three or less, the interleaved code can correct all bursts of length $3i$ or less. Thus, the codes are optimal burst-correctors in the sense that the equality holds in the Reiger bound,[1] i.e.,

$$b = \frac{n - k}{2}$$

where b is the guaranteed burst-correcting ability of the code, n is the code length, and k is the number of information symbols in the code.

In each case, the i subwords were decoded independently using the Peterson algorithm.[2] Decoding in this manner, rather than using the Peterson algorithm to decode the cyclic code of length $15i$ directly, enables the code to correct many error patterns which would otherwise be uncorrectable. This improves performance considerably since it enables the code to correct most error patterns of low weight $(2,3,4, \cdots, 10$, say) even thougb its minimum distance is only 3. It also simplifies the decoding circuitry somewhat. In this case the decoder employed approximately 210 transistors and a $14(i - 1)$-bit, limited-access, line-speed storage device.

The data are summarized in Table I. Because there does not seem to exist a single adequate measure of performance for such systems, the

* *Data-Phone* is a service mark of the Bell System.

TABLE I — SUMMARY OF DATA

Interleaving degree, i	73	200
Code length, $n = 15i$	1095	3000
Number of information symbols, $k = 9i$	657	1800
Burst-correcting ability, $b = (n - k)/2$	219	600
Number of calls	126	279
Number of hours	259	284
Number of bits transmitted	$1.9 \cdot 10^{9}$	$2.0 \cdot 10^{9}$
Line bit error rate	$5.7 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$
Number of line errors*	3613	5171
Number of delivered errors*	52	24
Improvement factor	70	215
Mean time between delivered errors* (hours)	5.0	11.9
Delivered error rate (errors* per bit)	$2.7 \cdot 10^{-8}$	$1.2 \cdot 10^{-8}$
Number of line word errors	3972	8704
Number of delivered word errors	83	59
Word improvement factor	48	150
Delivered word error rate (word errors per word)	$4.8 \cdot 10^{-5}$	$8.7 \cdot 10^{-5}$
Number of line bit errors	10607	24472
Number of delivered bit errors	2109	1703
Bit improvement factor	5	14
Delivered bit error rate (bit errors per bit)	$1.1 \cdot 10^{-6}$	$8.5 \cdot 10^{-7}$

results are presented in terms of three different types of error rate. These are based on bit errors, n-bit word errors, and errors.* The utility of the first two performance measures is apparent; however, in many situations the third is the most appropriate. For example, the mean time between errors* is the average duration of error-free intervals of useful length. This is a meaningful figure-of-merit for data users who require perfect transmission almost all of the time and for whom the cost of an error is relatively insensitive to the duration or the bit-error-density of the error. It is of interest to note that, regardless of how measured, the improvement in performance attributable to the error control system appears to increase approximately linearly with the degree of interleaving. Also the average line bit error rate encountered in this test was quite close to values reported in similar experiments on telephone facilities.

The author wishes to thank the following individuals for their cooperation throughout the course of the experiment: G. S. Robinson, who built the original error-control system (with $i = 73$); P. Mecklenburg, who changed the interleaving degree to 200 and suggested several

* An error is defined as a sequence of words all of which contain at least one bit error.

improvements in the system; and A. R. Lingenfelter, who was responsible for recording and reducing the data.

REFERENCES

1. Reiger, S. H., Codes for the Correction of "Clustered" Errors, IRE Trans., *IT6*, 1950, pp. 16–21.
2. Peterson, W. W., *Error-Correcting Codes*, MIT Press, 1961, p. 189.

A Note on a Type of Optimization Problem that Arises in Communication Theory

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A problem that has arisen[1] in connection with the use of transversal filters to reduce the effect of intersymbol interference in digital communication systems is to determine a real N-vector $c \triangleq (c_1, c_2, \cdots, c_N)$ such that, with $n_0 \, \varepsilon \, \mathfrak{F} \triangleq \{1,2, \cdots, N\}$,

$$\sum_{\substack{n=-\infty \\ n \neq n_0}}^{\infty} \left| \sum_{j \varepsilon \mathfrak{F}} c_j x_{n-j} \right| \tag{1}$$

is minimized subject to the constraint

$$1 = \sum_{j \varepsilon \mathfrak{F}} c_j x_{n_0 - j} . \tag{2}$$

Here $\{x_n\}_{-\infty}^{\infty}$ denotes a set of real constants such that $| x_0 | > \sum_{n \neq 0} | x_n |$. Lucky[1] has proved the interesting theorem that the optimal choice of c coincides with the unique solution[2] of the equations

$$\begin{aligned} 1 &= \sum_{j \varepsilon \mathfrak{F}} c_j x_{n_0 - j} \\ 0 &= \sum_{j \varepsilon \mathfrak{F}} c_j x_{n-j} , \qquad n \, \varepsilon \, \mathfrak{F} - \{n_0\}. \end{aligned} \tag{3}$$

The proof of Ref. 1 consists of establishing a contradiction to the assertion that (1), with c_{n_0} eliminated with the aid of (2), is minimized for some c for which (3) is not satisfied. The reader is referred to Ref. 1 for the details.

The purpose of this note is to show that Lucky's result, and far more general results of similar type, can be directly deduced from the following proposition.